

The Equipartition Magnetic Field Formula in Starburst Galaxies: Accounting for Pionic Secondaries and Strong Energy Losses

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ABSTRACT

Equipartition arguments provide an easy way to find a characteristic scale for the magnetic field from radio emission, by assuming the energy densities in cosmic rays and magnetic fields are the same. Yet most of the cosmic ray content in star-forming galaxies is in protons, which are invisible in radio emission. Therefore, the argument needs assumptions about the proton spectrum, typically that of a constant proton/electron ratio. In some environments, particularly starburst galaxies, the reasoning behind these assumptions does not necessarily hold: secondary pionic positrons and electrons may be responsible for most of the radio emission, and strong energy losses can alter the proton/electron ratio. We derive an equipartition expression that should work in a hadronic loss-dominated environment like starburst galaxies. Surprisingly, despite the radically different assumptions from the classical equipartition formula, numerically the results for starburst magnetic fields are similar. We explain this fortuitous coincidence using the energetics of secondary production and energy loss times. We show that these processes cause the proton/electron ratio to be ~ 100 for GHz-emitting electrons in starbursts.

Key words: galaxies: magnetic fields – galaxies: starburst – radio continuum: general

1 INTRODUCTION

Magnetic fields are important in astrophysics, but in practice their strengths are very hard to directly determine. Their presence is inferred through synchrotron emission of cosmic ray (CR) electrons and positrons (e^\pm) gyrating in magnetic fields. Synchrotron emission is detected from star-forming galaxies, demonstrating that they have magnetic fields (e.g., Condon 1992; Beck 2005). However, the synchrotron emission only informs us of a combination of cosmic ray content and the magnetic field content. In a few cases for star-forming galaxies, a detected gamma-ray spectrum provides additional evidence of the CR energy density (Aharonian et al. 2006; Abdo et al. 2010 and references therein), which when combined with plausible assumptions and modelling, allows us to constrain the magnetic fields in galaxies (e.g., de Cea del Pozo et al. 2009a; Crocker et al. 2010, 2011; Lacki & Thompson 2013). The great majority of galaxies are undetected in gamma rays, though (some upper limits are given in Lenain & Walter 2011 and Ackermann et al. 2012). Faraday measurements, which require a polarization signal, are a useful way of measuring ordered magnetic field strengths. Unfortunately, starbursts likely have highly turbulent magnetic fields, so the expected polarization is small. Anisotropy introduced in the turbulence by shearing and compression (Laing 1980; Sokoloff et al. 1998; Beck 2012) may result in some polarization signal (c.f. Greaves et al. 2000; Jones 2000), but there still will be no Faraday signal (e.g., Beck 2005). Furthermore in starbursts, Faraday depolarization can remove any polarization signal at typical observing frequencies (Reuter et al. 1994; Sokoloff et al. 1998). Faraday rotation measures also can be biased if there are relationships between the magnetic field strength and density, or if anisotropic turbulence is present on the sightline (Beck et al. 2003).

The equipartition and minimum energy arguments are common ways of finding a characteristic magnetic field strength B (e.g., Beck 2001). For a given radio luminosity, if B is very small, then the CR energy density U_{CR} must be very large; likewise, if U_{CR} is very small, then B must be very high. In between, there is a single magnetic field strength where $U_B = B^2/(8\pi)$

is equal to U_{CR} : this is the equipartition magnetic field strength (B_{eq}). Alternatively, for a given radio flux, the combined nonthermal energy density $U_B + U_{\text{CR}}$ has a minimum at a magnetic field strength B_{min} which is of the same order as (though generally distinct from¹) B_{eq} (Burbidge 1956a).

Throughout much of the Milky Way, equipartition between the magnetic fields and CRs holds (e.g., Niklas & Beck 1997). Likewise, in other normal star-forming galaxies, equipartition likely holds to within a factor of ~ 10 in B (e.g., Duric 1990). It is less clear if equipartition holds in starbursts, although any deviation would be interesting in its own right for the propagation of CRs and the sources of the magnetic fields (e.g., Thompson et al. 2006; Lacki, Thompson, & Quataert 2010). Equipartition has been proposed as a cause of the observed correlation between the far-infrared and radio luminosities of star-forming galaxies (Niklas & Beck 1997). On the other hand, Thompson et al. (2006) argued that equipartition (between magnetic fields and CRs) predicts magnetic field strengths too low to allow starbursts to lie on the correlation, since other radiative losses are extremely fast and B must be large for there to be any significant synchrotron emission (see also Lacki et al. 2010). However, the ease of equipartition methods – compared to methods involving gamma rays, detailed spectral modelling, or polarization methods – has led to their application in a variety of environments including normal galaxies, the Galactic Center (e.g., LaRosa et al. 2005; Ferrière 2009), starburst galaxies (e.g., Völk, Klein, & Wielebinski 1989; Beck et al. 2005; Persic & Rephaeli 2010; Beck 2012), and galaxies at high redshift (Murphy 2009; Chakraborti et al. 2012).

A problem with equipartition-style estimates is that most of the CR energy density is actually invisible in synchrotron emission: CR protons (and nuclei) are the dominant CR population at high energies. Only in the few cases where gamma-ray observations are available can the proton energy density be constrained directly (Acero et al. 2009; Acciari et al. 2009; Lacki et al. 2011; Persic & Rephaeli 2012). Radio equipartition estimates therefore make assumptions about how to convert the observed CR electron population in some observed frequency range into the total CR proton energy density. The simplest and most common assumption is to assume that the CR proton and electron spectra are power laws in energy with the same spectral index, and that there is a single ratio $\kappa = 30 - 100$ that sets the ratio for all galaxies. This is likely to be true for the injection spectra of primary CRs (Bell 1978; Schlickeiser 2002), at least those associated with star formation. The spectrum of Milky Way CRs indicates the primary injection proton/electron is near ~ 100 (Ginzburg & Ptuskin 1976).

The problem with this assumption is that the CR spectrum can be complex, especially for CR e^\pm . Roughly speaking, the steady-state CR spectrum $N(E)$ is equal to the product of the injection spectrum $Q(E)$ and the characteristic loss (cooling or escape) time $t_{\text{loss}}(E)$. When CR protons and electrons are governed by the same losses, such as diffusive escape in normal galaxies, the proton/electron ratio is preserved. In starburst galaxies, however, the CRs are likely to experience a variety of energy losses with different energy dependencies, which can alter the actual proton/electron ratio (Beck & Krause 2005). At high energies, e^\pm are cooled quickly by synchrotron and Inverse Compton (IC) losses, which increases the proton/electron ratio with energy. This is seen in the Milky Way at energies $\gtrsim 10$ GeV (see for example Figure 3.29 of Schlickeiser 2002). In addition, there are ionization losses for both e^\pm and protons, bremsstrahlung losses of e^\pm , and winds, which can alter the CR e^\pm spectrum (Thompson et al. 2006; Murphy 2009; Lacki et al. 2010).

A further complication is the possible presence of pionic secondary e^\pm in starburst galaxies. Secondary e^\pm are generated when protons crash into ambient gas atoms, creating unstable pions that decay into gamma rays, neutrinos, and e^\pm . In the dense gases of starbursts, the amount of secondaries may be comparable to or even dominant over the primary electrons (e.g., Torres 2004; Rengarajan 2005; Thompson, Quataert, & Waxman 2007; Lacki et al. 2010).

In this paper, we present equipartition and minimum energy formulae that should work in starburst environments. These take into account the presence of secondary e^\pm and the strong energy losses of starburst galaxies.

2 DERIVATION OF EQUIPARTITION AND MINIMUM ENERGY MAGNETIC FIELDS IN STARBURSTS

2.1 Basic assumptions

The spectrum $N(E)$ of CRs is governed by the diffusion-loss equation:

$$\frac{\partial N(E)}{\partial t} = Q(E) + \frac{d}{dE}[b(E)N(E)] - \frac{N(E)}{\tau(E)} + D\nabla^2 N(E) \quad (1)$$

as given in Torres (2004). Here $Q(E)$ is the injection spectrum of CRs, $b(E) = -dE/dt$ is the cooling rate of individual CRs, $\tau(E)$ is the loss time to escape and losses that remove most of the CR's energy in one interaction (catastrophic losses), and D is the spatial diffusion constant. We can simplify the equation to the leaky box equation if we assume that the modelled region is homogeneous, so that all spatial terms drop out, and is in equilibrium ($\partial N(E)/\partial t = 0$). The leaky box equation is then

¹ When neither CR protons nor CR e^\pm are cooled, the minimum energy and equipartition magnetic field strengths are equal if CRs are injected with a spectral index of 3 (Beck & Krause 2005).

$$0 = Q(E) + \frac{d}{dE}[b(E)N(E)] - \frac{N(E)}{\tau(E)} \quad (2)$$

where escape out of the region is now considered a catastrophic loss.

In the test particle approach to CR acceleration, CRs are injected with a momentum power law spectrum (Bell 1978). To simplify matters, we assume that CRs are injected with an energy power law spectrum:

$$Q(E) = Q_0 \left(\frac{E}{mc^2} \right)^{-p}, \quad (3)$$

where E is the total (rest plus kinetic) energy of the particle. The power law spectrum continues to a maximum energy $\gamma_{\max} mc^2$.

At high energies, protons escape or lose energy through pion production. Since pion production can be modelled as a catastrophic loss (Torres 2004), the proton spectrum is simply

$$N_p(E) = Q_p(E)\tau_p(E). \quad (4)$$

The proton lifetime in starbursts τ_p is thought to be dominated by advection or possibly pionic losses in denser starbursts like Arp 220. The fact that starbursts are observed at TeV energies (Acero et al. 2009; Acciari et al. 2009) indicates hard gamma-ray spectra ($\Gamma \approx 2.2$), supporting the idea that τ_p is set by one of these energy-independent processes. The gamma-ray observations of M 82 and NGC 253 suggest that more CR proton power is advected away in their nuclear starburst winds than is lost to pion production, although the efficiency of pionic losses is still much greater than in the Milky Way (Lacki et al. 2011). As we will see, though, it is convenient to scale τ_p with the pionic lifetime t_π , because the secondary e^\pm injection rate is directly tied to the pionic loss time. We therefore parameterize the proton lifetime as:

$$\tau_p = F_{\text{cal}} t_\pi. \quad (5)$$

CR e^\pm in contrast are thought to largely be trapped in starburst galaxies (Völk 1989), although winds may be quick enough to remove e^\pm before they cool in some starbursts (Heesen et al. 2011). Their losses are therefore continuous. The total e^\pm energy loss rate $b(E)$ can also be written as a cooling time $t_e(E) = E/b(E)$, giving us a steady-state spectrum

$$N_e(E) = \frac{Q_e(E)E}{(p-1)b(E)} = \frac{Q_e(E)t_e(E)}{p-1}. \quad (6)$$

To get the total CR proton energy density, we integrate its kinetic energy over all allowed energies: $U_p = \int_{m_p c^2}^{\gamma_{\max} m_p c^2} N_p(E) K dE$. Substituting in the solutions for the steady-state and injection proton spectra, we find

$$U_p = t_\pi F_{\text{cal}} Q_0 (m_p c^2)^2 \Upsilon \quad (7)$$

where we define a factor

$$\Upsilon = \begin{cases} \frac{1 - \gamma_{\max}^{2-p}}{p-2} + \frac{1 - \gamma_{\max}^{1-p}}{p-1} & (p \neq 1, 2) \\ (\ln \gamma_{\max} + 1 - 1/\gamma_{\max}) & (p = 2) \end{cases} \quad (8)$$

that depends on the shape of the injection spectrum. Unless otherwise noted in the paper, we assume $\gamma_{\max} = \infty$ and parameterize p as $2.2 + \Delta p$, so that $\Upsilon = 4.1\bar{6} = 25/6$ when $\Delta p = 0$. We can compare this to the luminosity of protons at energy E , which can be approximated as $E^2 Q_p(E)$. We find

$$U_p = \left(\frac{E_p}{m_p c^2} \right)^{p-2} \Upsilon t_\pi F_{\text{cal}} \times E_p^2 Q_p(E) \quad (9)$$

2.2 Relating the secondary e^\pm to CR protons

Calculating the secondary e^\pm spectrum involves integrating the differential cross sections for pionic e^\pm production over the CR proton spectrum (e.g., Kamae et al. 2006; Kelner, Aharonian, & Bugayov 2006). However, at sufficiently high e^\pm energies, the secondary e^\pm spectrum can be estimated using a δ -function approximation. A fraction F_{cal} of the CR proton energy goes into pion production, with the rest either escaping or lost to ionization or Coulomb cooling. Charged pions makes up $\sim 2/3$ of the pions produced, with the remaining $1/3$ going into neutral pions that mainly decay into gamma rays. Assuming equal energy going into the pion decay products, approximately $1/4 \times 2/3 \approx 1/6$ of the power in pions ends up in secondary e^\pm (c.f., Burbidge 1956b; Loeb & Waxman 2006). The other half of the pionic luminosity ($3/4$ of the charged pion power) goes into neutrinos. For the δ -function approximation, we therefore have:

$$E_{\text{sec}}^2 Q_{\text{sec}}(E_{\text{sec}}) = \frac{F_{\text{cal}}}{6} E_p^2 Q_p(E_p) \quad (10)$$

Table 1. Values of g for synchrotron emission spectra

Loss process	$a \equiv \frac{d \ln t}{d \ln E}$	$g(p = 2.0)$	$g(p = 2.2)$	$g(p = 2.4)$
Synchrotron / Inverse Compton	-1.0	1.0	1.00018	1.0112
Bremsstrahlung	0.0	1.2343	1.14299	1.07964
Ionization	1.0	3.00492	2.26698	1.83161

Since the typical inelasticity per inelastic collision is 20 per cent, and each pion decays into four particles, the typical energy of a secondary e^\pm is roughly $E_{\text{sec}} \approx 0.05 E_p$.

Do secondaries in fact dominate the e^\pm spectra of starbursts? With equation 10, we can estimate the primary to secondary ratio:

$$\frac{Q_{\text{sec}}(E_e)}{Q_{\text{prim}}(E_e)} \approx K_0 \frac{F_{\text{cal}}}{6} \left(\frac{E_p}{E_e} \right)^{2-p} \quad (11)$$

where we define $K_0 = Q_p/Q_{\text{prim}}$ as the number ratio of injected primary protons and primary electrons at high energies ($\gtrsim m_p c^2$). While the injection rate of protons vastly overwhelms that of primary electrons at a constant energy, the secondary e^\pm are injected at much lower energy than their parent protons, where there is more power in primary electrons for $p > 2$. Thus the secondary e^\pm are ‘diluted’ with respect to the primary electrons (e.g., Lacki et al. 2010). For $p = 2.2$, we find

$$\frac{Q_{\text{sec}}(E_e)}{Q_{\text{prim}}(E_e)} \approx 6.9 F_{\text{cal}} \left(\frac{K_0}{75} \right). \quad (12)$$

In terms of the fraction of the total e^\pm population in secondaries, $f_{\text{sec}} = Q_{\text{sec}}/(Q_{\text{sec}} + Q_{\text{prim}}) = [1 + (Q_{\text{sec}}/Q_{\text{prim}})^{-1}]^{-1}$, or

$$f_{\text{sec}} \approx \left[1 + 0.15 F_{\text{cal}}^{-1} \left(\frac{K_0}{75} \right)^{-1} \right]^{-1} \quad (13)$$

for $p = 2.2$.

The gamma-ray spectra of M 82 and NGC 253 indicate that $F_{\text{cal}} \approx 0.2 - 0.5$ (Lacki et al. 2011), so our equations imply secondary to primary ratios of $\sim 1.4 - 3.4$, or $f_{\text{sec}} \approx 0.6 - 0.8$. This is in line with detailed modelling of these galaxies (Domingo-Santamaría & Torres 2005; de Cea del Pozo, Torres, & Rodriguez Marrero 2009b; Rephaeli, Arieli, & Persic 2010).

2.3 Relating the radio emission to the e^\pm spectrum

We also consider a δ -function approximation in calculating the synchrotron emission. In this approximation, we assume each e^\pm of energy E_e radiates all of its synchrotron emission at a frequency:

$$\nu = \frac{3E_e^2 e B \langle \sin \alpha \rangle}{4\pi m_e^3 c^5} = \frac{3E_e^2 e B}{16m_e^3 c^5}, \quad (14)$$

with a magnetic field strength B and an electron electric charge e (Rybicki & Lightman 1979). We have used the fact that the mean sine pitch angle $\langle \sin \alpha \rangle$ for an isotropic distribution of CR e^\pm is $\langle \sin \alpha \rangle = \pi/4$. While physically speaking the δ -function approximation is not correct since the synchrotron emission of an e^\pm is a broad continuum, it gives acceptably accurate results for a power law distribution of e^\pm energies (Felten & Morrison 1966).

Since an e^\pm would radiate its energy in a synchrotron lifetime t_{synch} , the volumetric synchrotron luminosity of a homogeneous region is $\nu \epsilon_\nu = d\epsilon/d \ln \nu = (dU_e/d \ln \nu)/t_{\text{synch}} = (E_e dN_e/d \ln \nu)/t_{\text{synch}}$, where ν is the observed frequency. From equation 14, $d \ln \nu = 2 d \ln E_e$, so we have $\nu \epsilon_\nu \approx 1/2 (E_e dN/d \ln E_e)/t_{\text{synch}} = 1/2 (E_e^2 N_e(E_e))/t_{\text{synch}}$. The steady-state spectrum of the electrons is given by equation 6; substituting that in, we get

$$\nu \epsilon_\nu = \frac{1}{2} E_e^2 Q_e(E_e) \frac{g t_e(E_e)}{(p-1) t_{\text{synch}}(E_e)}. \quad (15)$$

The factor g corrects our approximation for $\nu \epsilon_\nu$ to the exact result for a power law spectrum of ultrarelativistic e^\pm at the frequency ν_C (Rybicki & Lightman 1979). If the electron lifetime is dominated by a continuous energy loss process and scales as $t_e(E_e) \propto E_e^a$, then the value of g is:

$$g = \frac{27\sqrt{3}}{(p-a+1)} 2^{-3(5+a-p)/2} \pi^{(2+a-p)/2} \Gamma\left(\frac{5+p-a}{4}\right) \Gamma\left(\frac{3p-3a-1}{12}\right) \Gamma\left(\frac{19+3p-3a}{4}\right) \left[\Gamma\left(\frac{7+p-a}{4}\right) \right]^{-1}. \quad (16)$$

We compile the values of g for different loss processes and injection indices p in Table 1, but for bremsstrahlung, synchrotron, or IC cooling, g is very nearly 1, and it is exactly 1 when $p = 2.0$ and $a = -1.0$ (as in the case when synchrotron cooling sets the e^\pm lifetime).

Now we can solve for the proton spectrum given a synchrotron volumetric luminosity. Combining equations 9, 10, and 15, we have

$$U_p = 12g^{-1} \left(\frac{E_p}{m_p c^2} \right)^{p-2} (p-1) \Upsilon \frac{t_\pi}{t_e(E_e)} t_{\text{synch}}(E_e) f_{\text{sec}} \nu \epsilon_\nu. \quad (17)$$

The pitch-angle averaged synchrotron cooling time is

$$t_{\text{synch}} = \frac{6\pi(m_e c^2)^2}{\sigma_T c E_e B^2}, \quad (18)$$

where σ_T is the Thomson cross section (Rybicki & Lightman 1979). Substituting it into eqn. 17, and using eqn. 14 to put E_e in terms of frequency and magnetic field, we find that

$$U_p = \frac{72\pi m_e c^2}{g \sigma_T c} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c \nu}{3e} \right)^{(p-3)/2} B^{-(p+1)/2} (p-1) \Upsilon \frac{t_\pi}{t_e(E_e)} f_{\text{sec}} \nu \epsilon_\nu \quad (19)$$

2.4 Which loss process is most important?

A key ingredient that we need to derive the equipartition formula is the e^\pm cooling time. Losses from synchrotron, Inverse Compton (IC), bremsstrahlung, ionization, and advection all potentially contribute. The e^\pm lifetime from all of these processes is:

$$t_e = [t_{\text{synch}}^{-1} + t_{\text{IC}}^{-1} + t_{\text{brems}}^{-1} + t_{\text{ion}}^{-1} + t_{\text{wind}}^{-1}]^{-1}. \quad (20)$$

Our discussion here is similar to that of Thompson et al. (2006), Murphy (2009), and Lacki et al. (2010). We consider M 82 as an example (the other nearby starburst NGC 253 has a similar environment), which has a magnetic field strength of $\sim 200 \mu\text{G}$ (from detailed models of the radio spectrum; Domingo-Santamaría & Torres 2005; Persic, Rephaeli, & Arieli 2008; de Cea del Pozo et al. 2009b; Rephaeli et al. 2010), a mean gas density of $\sim 300 \text{ cm}^{-3}$ (Weiß et al. 2001, assuming a scale height of 50 pc), and a radiation energy density of $\sim 1000 \text{ eV cm}^{-3}$ (Sanders et al. 2003, again assuming a starburst radius of 250 pc). Of course, none of these values are constant throughout the region, and it is not known how inhomogeneity affects CRs in starbursts, but using the average values seems to work for models fitting current radio and gamma-ray data (de Cea del Pozo et al. 2009a). After using eqn. 14 to convert between e^\pm energy to observed frequency, the synchrotron lifetime of GHz-emitting e^\pm is

$$t_{\text{synch}} = 498 \text{ kyr} \left(\frac{B}{200 \mu\text{G}} \right)^{-3/2} \left(\frac{\nu}{1 \text{ GHz}} \right)^{-1/2}, \quad (21)$$

from equation 18. For a radiation energy density U_{rad} the IC lifetime is $t_{\text{IC}} = 3\sqrt{3}/(16\sigma_T U_{\text{rad}}) \sqrt{m_e c e B / \nu}$ (Rybicki & Lightman 1979):

$$t_{\text{IC}} = 495 \text{ kyr} \left(\frac{B}{200 \mu\text{G}} \right)^{1/2} \left(\frac{\nu}{1 \text{ GHz}} \right)^{-1/2} \left(\frac{U_{\text{rad}}}{1000 \text{ eV cm}^{-3}} \right)^{-1} \quad (22)$$

The bremsstrahlung lifetime is only very weakly energy dependent, and at high e^\pm energies it is simply

$$t_{\text{brems}} = 104 \text{ kyr} \left(\frac{n_H}{300 \text{ cm}^{-3}} \right)^{-1} \quad (23)$$

where we assume that $n_H = 10n_{\text{He}}$ in the interstellar medium (Strong & Moskalenko 1998). The ionization lifetime is $t_{\text{ion}} = \gamma/[2.7c\sigma_T(6.85 + 0.5 \ln \gamma)n_H]$, where $\gamma = E_e/(m_e c^2) = \sqrt{16m_e c \nu / (3eB)}$ is the e^\pm Lorentz factor, and again assuming that $n_H = 10n_{\text{He}}$ (Schlickeiser 2002). If we evaluate the $\ln \gamma$ term as $\ln 1000$, since 1000 is the approximate Lorentz factor of GHz-emitting e^\pm in starbursts, then we find

$$t_{\text{ion}} \approx 178 \text{ kyr} \left(\frac{B}{200 \mu\text{G}} \right)^{-1/2} \left(\frac{\nu}{1 \text{ GHz}} \right)^{1/2} \left(\frac{n}{300 \text{ cm}^{-3}} \right)^{-1}. \quad (24)$$

Finally the advection time is roughly the time it takes to cross one scale height h at the wind speed v_{wind} , with wind speeds of order a few hundred km s^{-1} observed in M 82 (e.g., Greve 2004; Westmoquette et al. 2009).² Taking $h = 50 \text{ pc}$ and $v_{\text{wind}} = 300 \text{ km s}^{-1}$, we find

$$t_{\text{wind}} = 163 \text{ kyr} \left(\frac{h}{50 \text{ pc}} \right) \left(\frac{v_{\text{wind}}}{300 \text{ km s}^{-1}} \right)^{-1}. \quad (25)$$

² We assume that both the gas and the synchrotron-emitting CRs have the same scale height h , since secondary e^\pm are created by interactions of CR protons with gas and the CR e^\pm lifetimes are short in starbursts.

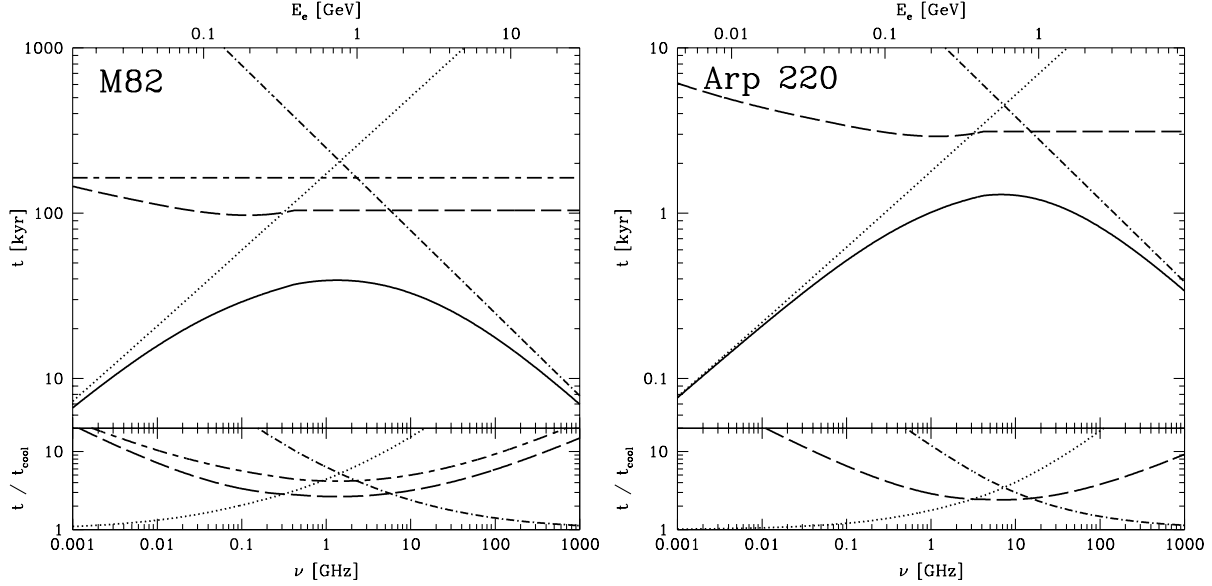


Figure 1. Comparison of the e^\pm loss times from different processes in an M 82-like starburst (left) and in an Arp 220-like starburst (right). The total loss time t_{cool} is the solid line, while bremsstrahlung is long-dashed, ionization is dotted, wind losses are long/short-dashed, and synchrotron and IC together are dash-dotted. On the top are the absolute loss times, while on the bottom is the ratio of each individual loss time to the total loss time.

For our fiducial values of the physical parameters in M 82, we see that bremsstrahlung is the most important cooling process for 1 GHz emitting e^\pm , though ionization and winds are also important. However, the final e^\pm lifetime is $t_{\text{cool}} \approx 39$ kyr, shorter than even the bremsstrahlung losses. We can quantify the importance of other losses relative to bremsstrahlung with the ratio

$$X = \frac{t_{\text{brems}}}{t_e}. \quad (26)$$

For our fiducial M 82 values, $X = 2.6$ at 1 GHz; other losses are even less important.

A comparison of how the individual loss times compare to each other at different frequencies is shown in Figure 1 (left). In this figure, we include the $\ln \gamma$ dependence in the ionization loss formula, and we use the full bremsstrahlung losses as given in Strong & Moskalenko (1998).³ Bremsstrahlung does dominate the losses of e^\pm that emit synchrotron between 300 MHz and 6 GHz, with $X \sim 3$ in this range. At lower frequencies, ionization is the most important loss, while at higher frequencies synchrotron and IC are the most important losses.

We also consider the radio nuclei of Arp 220, using a magnetic field ~ 2 mG (e.g., Lacki et al. 2010), a density of $\sim 10^4 \text{ cm}^{-3}$ (Downes & Solomon 1998), and a radiation energy density equivalent to that of a 50 K blackbody ($\sim 30000 \text{ eV cm}^{-3}$). The loss times for these conditions are shown in the right panel of Figure 1. At 1 GHz, ionization now is the most important loss: this is because we see lower energy e^\pm with the higher magnetic field strength. Near 10 GHz, though, bremsstrahlung again is the strongest loss, with $X \sim 3$.

For other starbursts, we generally expect advection, bremsstrahlung, or ionization losses to dominate: advection in low density starbursts (c.f., Crocker et al. 2011), ionization in starbursts with high magnetic fields (likely in the densest starbursts like Arp 220) where we see low energy e^\pm , and bremsstrahlung in intermediate density starbursts. At frequencies where bremsstrahlung dominates, $X \approx 3$ (compare with Figure 4, left panel of Lacki et al. 2010). In starbursts where advection is very strong, however, protons are likely to escape rather than produce pionic secondary e^\pm . Since advection is equally strong for protons and e^\pm , the proton to electron ratio is just K_0 when primaries dominate the e^\pm spectrum, and the revised equipartition formulae of Beck & Krause (2005) apply.

2.5 The equipartition and minimum energy formulae

For the purposes of this paper, we assume that the CR energy density entirely consists of protons ($U_{\text{CR}} = U_p$). Because there are several different energy regimes governed by different loss processes, the e^\pm spectrum is complex and likely has intrinsic

³ The $\gamma \lesssim 100$ formula in equation C12 of Strong & Moskalenko (1998) is erroneously multiplied by an extra $m_e c^2$, as can be seen in Schlickeiser (2002), equation 4.4.17. We have corrected this when making Figure 1.

spectral curvature; accounting for all these loss processes would require knowledge of the density, radiation energy density, advection speed, diffusion constant, and magnetic field strength in the volume. More importantly, because there are always at least bremsstrahlung losses, and since $t_{\text{brems}} < t_{\pi}$, the steady-state proton/electron ratio will always be greater than the injection proton/electron ratio (c.f. equations 6 and 4). The injection electron/proton ratio is

$$\frac{Q_e(\text{prim} + \text{sec})}{Q_p} < \frac{1}{6} \left(\frac{E_p}{E_{\text{sec}}} \right)^{2-p} + \frac{1}{K_0}, \quad (27)$$

or $\lesssim 18$ per cent for $p = 2.0$, $\lesssim 11$ per cent for $p = 2.2$, and $\lesssim 7$ per cent for $p = 2.4$. We therefore conclude that considering only protons leads to an error of less than ~ 20 per cent in U_p . The other uncertainties are generally larger than this.

Another possible worry of assuming that $U_{\text{CR}} = U_p$ is that the CRs include heavier nuclei, especially helium. In the Milky Way, though, the hydrogen/helium ratio at equal rigidities is ~ 8 , and at equal energies per nucleon, the hydrogen/helium ratio is ~ 24 (Webber & Lezniak 1974; Webber, Golden, & Stephens 1987). This ratio must be multiplied by the atomic mass A of helium when calculating the energy density, but still helium is only $(1/24 - 1/8) \times 4 \approx 1/6 - 1/2$ of the CR energy density. Analysis of the CR flux at Earth indicate that helium makes up about $\sim 1/6$ of the Galactic CR energy density, with another $\sim 1/12$ from heavier nuclei (Webber 1998). Furthermore, helium produces pionic secondary e^{\pm} just as hydrogen does: as far as collisions go, a helium nucleus at high energies is much like a collection of protons with the same energy per nucleon. Nucleons can shield each other in heavy nuclei, reducing the pp cross section per nucleon as $A^{-1/4}$ for an atomic mass A (Orth & Buffington 1976; Strong & Moskalenko 1998), but for helium this just reduces the cross section by 30 per cent. Therefore, the hadronic starburst radio emission traces high energy CR helium nuclei as well as CR protons, and our derived U_p basically includes the contribution from helium as well.

The volumetric luminosity $\nu\epsilon_{\nu}$ can be found from either the total luminosity νL_{ν} or the intensity on a line of sight νI_{ν} . In the absence of absorption, the volumetric luminosity is simply:

$$\nu\epsilon_{\nu} = \nu L_{\nu} / V \quad (28)$$

where V is the volume of the radio wave emitting region. If the starburst is inhomogeneous, it is preferable to use the intensity instead of luminosity to calculate the equipartition field (Beck & Krause 2005). The (unabsorbed) intensity on a line of sight is given by $dI/d\ell = \epsilon_{\nu}/(4\pi)$, so that by assuming a constant ϵ_{ν} , we can replace the volumetric luminosity with:

$$\nu\epsilon_{\nu} = 4\pi\nu I_{\nu} / \ell \quad (29)$$

where ℓ is the sightline through the radio emitting region.

2.5.1 Bremsstrahlung-scaled losses

In general, the lifetime of the CR e^{\pm} is complex, depending on many loss processes. It is often convenient to scale the lifetime to the bremsstrahlung cooling lifetime. At GHz frequencies, bremsstrahlung is expected to be an important or even dominant cooling process for CR e^{\pm} in starburst galaxies (section 2.4). Furthermore, both pionic and bremsstrahlung losses are mostly independent of energy, both scale with density, and both are very nearly of the same magnitude:

$$t_{\pi} \approx 50 \text{ Myr} (n/\text{cm}^{-3})^{-1} \quad (30)$$

from Mannheim & Schlickeiser (1994) for the pionic loss time, and (c.f. equation 23)

$$t_{\text{brems}} \approx 31 \text{ Myr} (n/\text{cm}^{-3})^{-1} \quad (31)$$

for the bremsstrahlung lifetime.

Equipartition formula – By setting $B^2/(8\pi) = \zeta U_p = \zeta U_{\text{CR}}$, the equipartition CR energy density is then

$$U_{\text{CR}} = \left[\frac{9m_e c^2}{g\sigma_T c} (8\pi)^{(3-p)/4} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{X t_{\pi}}{t_{\text{brems}}} f_{\text{sec}} \nu \epsilon_{\nu} \zeta^{-(p+1)/4} \right]^{4/(5+p)} \quad (32)$$

with a magnetic field of

$$B_{\text{eq}} = \left[\frac{576\pi^2 m_e c^2}{g\sigma_T c} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{X t_{\pi}}{t_{\text{brems}}} f_{\text{sec}} \nu \epsilon_{\nu} \zeta \right]^{2/(5+p)} \quad (33)$$

The ratio $\zeta = U_B/U_{\text{CR}}$ allows one to consider departures from exact equipartition. Here, X is the ratio of bremsstrahlung cooling time to t_{cool} , as previously defined. Note that this assumes that X is constant in energy. However, X is likely to have some energy dependence, at least at energies far away from ~ 1 GeV. Since the frequency is a function of both energy and magnetic field, the magnetic field strength would enter implicitly into X ; in order to solve self-consistently for B , the energy dependence of X must be included when solving eqn. 19. In practice, because of the weak dependence of B_{eq} on X

Table 2. Ratio of equipartition and minimum energy magnetic fields

Formula	$\frac{B_{\text{eq}}}{B_{\text{min}}}$	$\frac{B_{\text{eq}}}{B_{\text{min}}}(p = 2.0)$	$\frac{B_{\text{eq}}}{B_{\text{min}}}(p = 2.2)$	$\frac{B_{\text{eq}}}{B_{\text{min}}}(p = 2.4)$	Reference
Classical formula	$[4/3]^{2/7}$	1.09	Beck & Krause (2005)
Revised formula	$[4/(p+1)]^{2/(p+5)}$	1.09	1.06	1.04	Beck & Krause (2005)
Hadronic formulae					
Bremsstrahlung-scaled losses	$[4/(p+1)]^{2/(p+5)}$	1.09	1.06	1.04	§ 2.5.1 (Eqns. 33 & 36)
Ionization-dominant losses	$[4/p]^{2/(p+4)}$	1.26	1.22	1.17	§ 2.5.2 (Eqns. 42 & 45)
Synchrotron-dominant losses	$[4/(p-2)]^{2/(p+2)}$	∞^a	4.2	2.8	§ 2.5.3 (Eqns. 49 & 52)
IC-dominant losses	$[4/(p+2)]^{2/(p+6)}$	1.0	0.99	0.98	§ 2.5.4 (Eqns. 58 & 61)

^a: As discussed in Section 2.5.3, in practice this limit can never be attained, because of breaks in the CR spectrum, non steady-state effects, and other radiative losses that become important when B is low.

($B \propto X^{2/7.2}$), the effects of energy dependence in X on the estimated equipartition magnetic field will be small. Furthermore, below the \sim GeV energies probed by GHz observations, $dX/dE < 0$ because ionization losses are quicker at low energies, and above \sim GeV, $dX/dE > 0$ because Inverse Compton and synchrotron losses are quicker at high energies, so we roughly expect $dX/dE \approx 0$ (as shown in Figure 1).

Suppose that the starburst is a homogeneous disc, and the synchrotron-emitting particles fill a fraction $f_{\text{fill}}^{\text{synch}}$ of that disk. If it has radius R and midplane-to-edge scale height h , then the emitting volume is $V = 2\pi R^2 h f_{\text{fill}}^{\text{CR}}$. The observed flux density S_ν is $L_\nu/(4\pi D^2)$, where D is the distance. Scaling to values typical of nearby starbursts like M 82 and NGC 253:

$$U_{\text{CR}} = 106 \text{ eV cm}^{-3} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{2.4+2\Delta p}{7.2+\Delta p}} \zeta^{-\frac{3.2+\Delta p}{7.2+\Delta p}} \times \left[23.4^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.14} \right)^{-1} \right]^{\frac{4}{7.2+\Delta p}} \quad (34)$$

$$B_{\text{eq}} = 65.4 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{1.2+\Delta p}{7.2+\Delta p}} \times \left[23.4^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \zeta \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.14} \right)^{-1} \right]^{\frac{2}{7.2+\Delta p}} \quad (35)$$

Minimum energy formula – The minimum energy magnetic field is defined as the magnetic field which minimizes $U_B + U_{\text{CR}}$ for the observed synchrotron luminosity. It is found by setting $d[B^2/(8\pi) + U_p]/dB = 0$. Numerically, the minimum energy estimate is very nearly the same as the equipartition estimate:

$$B_{\text{min}} = \left[(p+1) \frac{144\pi^2 m_e c^2}{g\sigma_T c} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{X t_\pi}{t_{\text{brems}}} f_{\text{sec}} \nu \epsilon_\nu \right]^{2/(p+5)} \quad (36)$$

$$B_{\text{min}} = 61.3 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{1.2+\Delta p}{7.2+\Delta p}} \times \left[24.2^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \left(1 + \frac{\Delta p}{3.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.14} \right)^{-1} \right]^{\frac{2}{7.2+\Delta p}} \quad (37)$$

The ratio $B_{\text{eq}}/B_{\text{min}}$ is $[4/(p+1)]^{2/(p+5)}$, the same as in the revised equipartition formula of Beck & Krause (2005) (see Table 2). When bremsstrahlung losses dominate, the proton to electron ratio is constant with energy, so the proton/electron ratio has the same behavior as if there are no losses.

2.5.2 Ionization-dominant losses

At low energies, ionization (or Coulomb losses) will always dominate the energy losses of CR e^\pm . For M 82, ionization losses should become most important for e^\pm radiating below ~ 300 MHz, while for Arp 220, ionization losses are most important all the way up to ~ 3 GHz (Figure 1). This assumption is therefore most appropriate for observations with low frequency telescopes like LOFAR when they observe starbursts. While more complicated to scale from pionic losses, because the ionization losses depend on electron energy, at sufficiently low energies they will be the only loss, making the calculation of the electron spectrum relatively simple.

Ignoring logarithmic terms in energy, the ionization loss time for CR e^\pm can be parameterized as:

$$t_{\text{ion}} = t_{\text{ion},0} \left(\frac{B}{B_0} \right)^{-1/2} \left(\frac{\nu}{\nu_0} \right)^{1/2} \left(\frac{n}{\text{cm}^{-3}} \right)^{-1}, \quad (38)$$

where $t_{\text{ion},0} = 53$ Myr for $B_0 = 200$ μG and $\nu_0 = 1$ GHz (compare equation 24). Likewise, the pionic loss time can be parametrized as:

$$t_\pi = t_{\pi,0} \left(\frac{n}{\text{cm}^{-3}} \right)^{-1} \quad (39)$$

This lets us cancel out the density dependence in the ionization and pionic loss times.

From equation 19, we have

$$U_p = \frac{72\pi m_e c^2}{g\sigma_{Tc}} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} B^{-(p+1)/2} (p-1) \Upsilon \frac{t_{\pi,0}}{t_{\text{ion},0}} \left(\frac{B}{B_0} \right)^{1/2} \left(\frac{\nu}{\nu_0} \right)^{-1/2} f_{\text{sec}} \nu \epsilon_\nu. \quad (40)$$

Equipartition formula – Solving for the equipartition magnetic field $B_{\text{eq}} = \sqrt{8\pi\zeta U_{\text{CR}}}$, we find

$$U_{\text{CR}} = \left[\frac{72\pi m_e c^2}{g\sigma_{Tc}} (8\pi)^{-p/4} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{t_{\pi,0}}{t_{\text{ion},0}} B_0^{-1/2} \left(\frac{\nu}{\nu_0} \right)^{-1/2} f_{\text{sec}} \nu \epsilon_\nu \zeta^{-p/4} \right]^{4/(p+4)}. \quad (41)$$

and

$$B_{\text{eq}} = \left[\frac{576\pi^2 m_e c^2}{g\sigma_{Tc}} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{t_{\pi,0}}{t_{\text{ion},0}} B_0^{-1/2} \left(\frac{\nu}{\nu_0} \right)^{-1/2} f_{\text{sec}} \nu \epsilon_\nu \zeta \right]^{2/(p+4)}. \quad (42)$$

For typical starburst values, we have

$$U_{\text{CR}} = 33.6 \text{ eV cm}^{-3} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{0.4+2\Delta p}{6.2+\Delta p}} \zeta^{-\frac{2.2+\Delta p}{6.2+\Delta p}} \times \left[31.2^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{2.27} \right)^{-1} \right]^{\frac{4}{6.2+\Delta p}} \quad (43)$$

and

$$B_{\text{eq}} = 36.8 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{0.2+\Delta p}{6.2+\Delta p}} \times \left[31.2^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \zeta \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{2.27} \right)^{-1} \right]^{\frac{2}{6.2+\Delta p}}. \quad (44)$$

Minimum energy formula – As with the bremsstrahlung loss formula, we solve for B_{min} by setting $d[B^2/(8\pi) + U_p]/dB = 0$.

We find

$$B_{\text{min}} = \left[p \frac{144\pi^2 m_e c^2}{g\sigma_{Tc}} \left(\frac{20m_e}{m_p} \right)^{p-2} \left(\frac{16m_e c\nu}{3e} \right)^{(p-3)/2} (p-1) \Upsilon \frac{t_{\pi,0}}{t_{\text{ion},0}} B_0^{-1/2} \left(\frac{\nu}{\nu_0} \right)^{-1/2} f_{\text{sec}} \nu \epsilon_\nu \right]^{2/(p+4)}. \quad (45)$$

Numerically, this comes out to

$$B_{\text{min}} = 30.4 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{0.2+\Delta p}{6.2+\Delta p}} \times \left[34.4^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \left(1 + \frac{\Delta p}{2.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{2.27} \right)^{-1} \right]^{\frac{2}{6.2+\Delta p}} \quad (46)$$

2.5.3 Synchrotron-dominant losses

At high energies, synchrotron and Inverse Compton losses always dominate the cooling of CR e^\pm . For M 82, this should happen for e^\pm emitting above ~ 5 GHz, and for Arp 220, the transition is for e^\pm emitting above ~ 15 GHz (Figure 1). In order for the observed infrared-radio correlation to hold for starbursts, it is thought that $U_B \approx U_{\text{rad}}$ in starbursts (Völk 1989; Condon et al. 1991; Lacki et al. 2010). Therefore for high frequency radio emission, it may be more convenient to scale to the synchrotron lifetime. This case was previously considered in Pfrommer & Enßlin (2004), but we rederive it with our more approximate approach to compare with the other formulae. Synchrotron losses do not depend on density, so the density dependence in t_π is not canceled out.

As we can see from equation 17, $t_{\text{cool}} = t_{\text{synch}}$ cancels the t_{synch} in the CR e^\pm emissivity. After converting the electron energy $E_e \approx E_p/20$ (where a pionic e^\pm typically has 1/20 the energy of its primary proton; section 2.2) into synchrotron frequency using eqn. 14, we are left with:

$$U_p = 12g^{-1} \left(\frac{16m_e c\nu}{3e} \right)^{(p-2)/2} B^{-(p-2)/2} \left(\frac{20m_e}{m_p} \right)^{p-2} (p-1) \Upsilon t_\pi f_{\text{sec}} \nu \epsilon_\nu \quad (47)$$

Equipartition estimate – Setting $B^2/(8\pi) = \zeta U_p = \zeta U_{\text{CR}}$, we find:

$$U_{\text{CR}} = \left[12g^{-1} (8\pi)^{-(p-2)/4} \left(\frac{16m_e c\nu}{3e} \right)^{(p-2)/2} \left(\frac{20m_e}{m_p} \right)^{p-2} (p-1) \Upsilon t_\pi f_{\text{sec}} \nu \epsilon_\nu \zeta^{-(p-2)/4} \right]^{4/(p+2)}. \quad (48)$$

The equipartition magnetic field strength is

$$B_{\text{eq}} = \left[96\pi g^{-1} \left(\frac{16m_e c\nu}{3e} \right)^{(p-2)/2} \left(\frac{20m_e}{m_p} \right)^{p-2} (p-1) \Upsilon t_\pi f_{\text{sec}} \nu \epsilon_\nu \zeta \right]^{2/(p+2)}. \quad (49)$$

Substituting in the previously used values, we find

$$U_{\text{CR}} = 5.45 \text{ eV cm}^{-3} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{4.4+2\Delta p}{4.2+\Delta p}} \zeta^{-\frac{0.2+\Delta p}{4.2+\Delta p}} \times \left[49.2^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}} \right)^{-1} \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.0} \right)^{-1} \right]^{\frac{4}{4.2+\Delta p}} \quad (50)$$

for the energy density and

$$B_{\text{eq}} = 14.8 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{2.2+\Delta p}{4.2+\Delta p}} \times \left[49.2^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \zeta \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}} \right)^{-1} \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.0} \right)^{-1} \right]^{\frac{2}{4.2+\Delta p}} \quad (51)$$

for the equipartition magnetic field strength.

The resulting energy densities and magnetic fields are much smaller than the bremsstrahlung estimates. This is because the synchrotron cooling alone is relatively slow compared to pionic cooling for the parameters used. Thus, the pionic e^\pm accumulate longer, leading to a smaller proton/electron ratio. With less ‘invisible’ CR proton content compared to CR e^\pm content, the energy density is smaller.

Minimum energy formula – The minimum energy magnetic field strength with these assumptions is

$$B_{\text{min}} = \left[24\pi g^{-1} (p-2) \left(\frac{16m_e c\nu}{3e} \right)^{(p-2)/2} \left(\frac{20m_e}{m_p} \right)^{p-2} (p-1) \Upsilon t_\pi f_{\text{sec}} \nu \epsilon_\nu \right]^{2/(p+2)}. \quad (52)$$

With our usual parameters, we find that B_{min} is much smaller than B_{eq} :

$$B_{\text{min}} = 3.56 \text{ } \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}} \right)^{\frac{2.2+\Delta p}{4.2+\Delta p}} \left[100^{\Delta p} \left(1 + \frac{\Delta p}{1.2} \right) \left(1 + \frac{\Delta p}{0.2} \right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}} \right) \left(\frac{R}{250 \text{ pc}} \right)^{-2} \left(\frac{h}{100 \text{ pc}} \right)^{-1} \right. \\ \left. \times \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}} \right)^{-1} \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.0} \right)^{-1} \right]^{\frac{2}{4.2+\Delta p}} \quad (53)$$

The reason that $B_{\text{min}} \ll B_{\text{eq}}$ (Table 2) is that the derived U_p depends very weakly on B . When all of the electron power is going into synchrotron, we have $\nu \epsilon_\nu = E_e^2 N_e(E_e)/t_{\text{synch}} = E_e^2 Q_e(E_e)t_{\text{synch}}/t_{\text{synch}} = E_e^2 Q(E_e)$. Even as B gets arbitrarily small, the assumption that e^\pm cool only by synchrotron implies that, in steady-state, the synchrotron luminosity is exactly equal to the injected power. The only way that B affects the synchrotron flux is the dependence of frequency on magnetic field strength: varying the magnetic field strength probes different parts of the e^\pm spectrum, where there are different amounts of power injected if $p \neq 2$. This is a weak effect. Therefore, since the derived U_p is nearly independent of B , the nonthermal energy density can usually be decreased by simply decreasing U_B with little effect on U_p . When $p = 2.0$ exactly, then B_{min} goes to 0. With a $p = 2$ spectrum, there is equal power injected at all energies, so the spectral effect disappears. In practice, this limit is not attained because (1) there is a high energy cutoff to the CR e^\pm spectrum, and the frequency it is observed at depends on B ; (2) steady-state may not be attained if synchrotron is the only radiative loss; and (3) as B decreases, the other losses become more important; if nothing else, when $B \leq 3.5(1+z)^2 \text{ } \mu\text{G}$, Inverse Compton losses from the CMB exceed

synchrotron losses, so this estimate is no longer valid. Finally, if synchrotron really were the only losses, then as $t_{\text{synch}} \rightarrow \infty$, U_e becomes bigger than U_p , invalidating our assumption that $U_{\text{CR}} \approx U_p$. In practice, this cannot happen for hadronic e^\pm because bremsstrahlung losses would always intervene (as discussed in the beginning of the section).

For comparison, Pfrommer & Enßlin (2004) found a minimum energy magnetic field strength for pionic e^\pm that can be written as

$$B_{\text{min}}^{P04} = \left[\frac{256\pi^2(p-2)(p+2)}{9\sqrt{3}\pi(p+10/3)} \frac{\mathcal{B}(\frac{p-2}{2}, \frac{3-p}{2})\Gamma(\frac{p+8}{4})}{\Gamma(\frac{3p+2}{12})\Gamma(\frac{3p+10}{12})\Gamma(\frac{p+6}{4})} \left(\frac{16m_e}{m_p}\right)^{p-2} \left(\frac{2\pi m_e c\nu}{3e}\right)^{(p-2)/2} \frac{\nu\epsilon_\nu}{n\sigma_{pp}c} \right]^{2/(p+2)} \quad (54)$$

when synchrotron losses dominate and $p \neq 2$, with σ_{pp} as the cross section for proton-proton pionic collisions (given in their equation 2.18), \mathcal{B} is the beta function, and Γ is the gamma function. This version of the estimate assumes there are no primary electrons ($f_{\text{sec}} = 1$). Plugging our fiducial parameters into this equation instead of equation 52 gives us $3.7 \mu\text{G}$ instead of $3.6 \mu\text{G}$ in eqn. 53. The result from our approach is therefore confirmed by Pfrommer & Enßlin (2004).

2.5.4 IC-dominant losses

The other dominant loss at high energies is Inverse Compton (IC). In starburst galaxies, IC losses are expected to be quick because of intense FIR radiation from star-forming regions (Condon et al. 1991); the only question is whether the other losses are stronger (see the discussion in 2.4; Völk 1989). The IC loss time has a convenient scaling with the synchrotron loss time:

$$t_{\text{IC}} = t_{\text{synch}}(U_B/U_{\text{rad}}) \quad (55)$$

where U_{rad} is the radiation energy density. We therefore have

$$U_p = 12g^{-1} \left(\frac{16m_e c\nu}{3e}\right)^{(p-2)/2} B^{-(p-2)/2} \left(\frac{20m_e}{m_p}\right)^{p-2} (p-1)\Upsilon t_\pi f_{\text{sec}} \frac{U_{\text{rad}}}{U_B} \nu\epsilon_\nu \quad (56)$$

These equipartition and minimum energy estimates require both a density and a radiation field.

Equipartition estimate – By setting the ratio of magnetic energy density to CR proton energy density as ζ , we find:

$$U_{\text{eq}} = \left[12g^{-1}(8\pi)^{-(p-2)/4} \left(\frac{16m_e c\nu}{3e}\right)^{(p-2)/2} \left(\frac{20m_e}{m_p}\right)^{p-2} (p-1)\Upsilon t_\pi f_{\text{sec}} U_{\text{rad}} \nu\epsilon_\nu \zeta^{-(p+2)/4} \right]^{4/(p+6)} \quad (57)$$

and

$$B_{\text{eq}} = \left[768\pi^2 g^{-1} \left(\frac{16m_e c\nu}{3e}\right)^{(p-2)/2} \left(\frac{20m_e}{m_p}\right)^{p-2} (p-1)\Upsilon t_\pi f_{\text{sec}} U_{\text{rad}} \nu\epsilon_\nu \zeta \right]^{2/(p+6)}, \quad (58)$$

with the accompanying scalings of:

$$U_{\text{CR}} = 69.3 \text{ eV cm}^{-3} \left(\frac{\nu}{1 \text{ GHz}}\right)^{\frac{4.4+2\Delta p}{8.2+\Delta p}} \zeta^{-\frac{4.2+\Delta p}{8.2+\Delta p}} \left[26.1^{\Delta p} \left(1 + \frac{\Delta p}{1.2}\right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}}\right) \left(\frac{R}{250 \text{ pc}}\right)^{-2} \left(\frac{h}{100 \text{ pc}}\right)^{-1} \right. \\ \left. \times \left(\frac{D}{3.5 \text{ Mpc}}\right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}}\right)^{-1} \left(\frac{U_{\text{rad}}}{1000 \text{ eV cm}^{-3}}\right) \left(\frac{\Upsilon}{25/6}\right) \left(\frac{g}{1.0}\right)^{-1} \right]^{\frac{4}{8.2+\Delta p}} \quad (59)$$

and

$$B_{\text{eq}} = 52.8 \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}}\right)^{\frac{2.2+\Delta p}{8.2+\Delta p}} \left[26.1^{\Delta p} \left(1 + \frac{\Delta p}{1.2}\right) \zeta \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}}\right) \left(\frac{R}{250 \text{ pc}}\right)^{-2} \left(\frac{h}{100 \text{ pc}}\right)^{-1} \right. \\ \left. \times \left(\frac{D}{3.5 \text{ Mpc}}\right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}}\right)^{-1} \left(\frac{U_{\text{rad}}}{1000 \text{ eV cm}^{-3}}\right) \left(\frac{\Upsilon}{25/6}\right) \left(\frac{g}{1.0}\right)^{-1} \right]^{\frac{2}{8.2+\Delta p}}. \quad (60)$$

Minimum energy estimate – Using eqn. 56 to find the minimum nonthermal energy density, we calculate

$$B_{\text{min}} = \left[(p+2)192\pi^2 \left(\frac{16m_e c\nu}{3e}\right)^{(p-2)/2} \left(\frac{20m_e}{m_p}\right)^{p-2} (p-1)\Upsilon t_\pi f_{\text{sec}} U_{\text{rad}} \nu\epsilon_\nu \right]^{2/(p+6)}, \quad (61)$$

or

$$B_{\text{min}} = 53.5 \mu\text{G} \left(\frac{\nu}{1 \text{ GHz}}\right)^{\frac{2.2+\Delta p}{8.2+\Delta p}} \left[25.9^{\Delta p} \left(1 + \frac{\Delta p}{1.2}\right) \left(1 + \frac{\Delta p}{4.2}\right) \frac{f_{\text{sec}} X}{f_{\text{fill}}^{\text{synch}}} \left(\frac{S_\nu}{\text{Jy}}\right) \left(\frac{R}{250 \text{ pc}}\right)^{-2} \left(\frac{h}{100 \text{ pc}}\right)^{-1} \right]$$

$$\times \left(\frac{D}{3.5 \text{ Mpc}} \right)^2 \left(\frac{n_H}{300 \text{ cm}^{-3}} \right)^{-1} \left(\frac{U_{\text{rad}}}{1000 \text{ eV cm}^{-3}} \right) \left(\frac{\Upsilon}{25/6} \right) \left(\frac{g}{1.0} \right)^{-1} \Bigg]^{\frac{2}{8.2+\Delta p}}. \quad (62)$$

for $p = 2.2 + \Delta p$. For p near 2, the minimum energy and equipartition magnetic field strengths are nearly identical in this case (Table 2).

Pfrommer & Enßlin (2004) also derived the minimum energy estimate for IC-loss dominated pionic e^\pm in a more sophisticated manner, finding:

$$B_{\text{min}}^{P04} = \left[\frac{2048\pi^3(p+2)^2}{9\sqrt{3}\pi(p+10/3)} \frac{\mathcal{B}(\frac{p-2}{2}, \frac{3-p}{2})\Gamma(\frac{p+8}{4})}{\Gamma(\frac{3p+2}{12})\Gamma(\frac{3p+10}{12})\Gamma(\frac{p+6}{4})} \left(\frac{16m_e}{m_p} \right)^{p-2} \left(\frac{2\pi m_e c \nu}{3e} \right)^{(p-2)/2} \frac{\nu \epsilon_\nu U_{\text{rad}}}{n \sigma_{pp} c} \right]^{2/(p+6)}. \quad (63)$$

Here, we have substituted the general radiation energy density U_{rad} for the CMB energy density. Plugging our fiducial starburst parameters into this equation gives us $55 \mu\text{G}$ as the coefficient for equation 62. Like the synchrotron-loss dominant formula, the results of Pfrommer & Enßlin (2004) are in accord with our own.

3 APPLYING THE EQUIPARTITION FORMULA

3.1 How much of the radio flux is diffuse and nonthermal?

So far, we have been assuming throughout the paper that all of a starburst's GHz radio flux is diffuse synchrotron emission from CR e^\pm in the interstellar medium. We also assumed that the synchrotron flux is transmitted freely to Earth. Neither is completely true, although it turns out these are often reasonable approximations.

At high frequencies, thermal free-free emission, which does not trace magnetic fields or CRs, becomes increasingly important, since it has a flat $\nu^{-0.1}$ spectrum. However, the total amount of free-free emission is limited by the number of ionizing photons generated by the starburst. At 1.4 GHz, the thermal fraction is at most $\sim 1/8$ for starbursts that lie on the observed infrared-radio correlation, as estimated by Condon (1992); if many ionizing photons escape the starburst or are destroyed by dust do not contribute, the thermal fraction might be smaller. A few very young starbursts appear to have no synchrotron flux, perhaps because supernovae have not gone off in them (Roussel et al. 2003), but most star-forming galaxies and starbursts have radio spectral indices $\ll -0.1$, indicating nonthermal emission dominates the GHz flux (e.g., Klein et al. 1988; Condon 1992; Niklas et al. 1997; Clemens et al. 2008; Ibar et al. 2010; Williams & Bower 2010). Radio spectrum fits to normal spiral galaxies indicate their typical 1.4 GHz thermal fraction is $\sim 10\%$ (Niklas, Klein, & Wielebinski 1997). For starburst galaxies, the spectral fits are more difficult, because the transition between the different cooling processes at different frequencies cause the intrinsic synchrotron spectrum to steepen (Thompson et al. 2006), whereas free-free emission flattens the spectrum; disentangling the effects of the two is difficult (Condon 1992). Model fits to starburst radio spectra so far generally indicate GHz thermal fractions of a few percent (e.g., Klein, Wielebinski, & Morsi 1988; Williams & Bower 2010). We adopt $f_{\text{therm}} = [9(\nu/\text{GHz})^{-0.6} + 1]^{-1}$, which is appropriate if the nonthermal synchrotron spectrum has a $\nu^{-0.7}$ spectrum and the thermal fraction at 1 GHz is 10%.

The same ionized matter that generates free-free emission must also have free-free absorption on some level, but the effects of absorption are harder to calculate because it depends on the geometry of the ionized gas. The simplest approach to estimate the free-free absorption from the radio spectrum is with uniform slab models, in which the synchrotron-emitting CRs and ionized 10^4 K gas are assumed to be homogeneous and fully mixed. With uniform slab models, the free-free turnovers are generally expected to be in the range of a few hundred MHz for M 82 and NGC 253's starburst cores, with optical depths of order ~ 0.1 at 1 GHz (Carilli 1996; Williams & Bower 2010; Adebahr et al. 2012). Free-free absorption may be important for frequencies up to a few GHz in Ultraluminous Infrared Galaxies (ULIRGs, Condon et al. 1991; Torres 2004; Clemens et al. 2008). In either case, free-free absorption might significantly suppress the observed synchrotron luminosity at frequencies where ionization cooling dominates.

Yet the ionized gas is not likely to be uniformly distributed in the starburst. Much of the ionized mass instead is likely concentrated discrete H II regions, which are dense but have small filling factor. Since not all sightlines through the starburst necessarily pass through an H II region, it is possible some synchrotron flux is transmitted even as the frequency descends deep into the MHz range (Lacki 2012). Radio recombination line observations are often interpreted with models of H II regions. These studies find H II regions of a wide range of densities in starbursts, but generally fall into the categories of relatively low density ($\sim 1000 \text{ cm}^{-3}$) regions with relatively high covering fractions and relatively high density ($\sim 10^5 \text{ cm}^{-3}$) regions with relatively low covering fractions (Anantharamaiah et al. 1993; Rodríguez-Rico et al. 2004; Rodríguez-Rico et al. 2006). In Arp 220, the high density H II regions are opaque even past 10 GHz, but they cover only a fraction of a percent of the starburst; a more diffuse component of ionized gas has an optical depth of $\sim 10\%$ at 8 GHz (Anantharamaiah et al. 2000).

Given the difficulty of accurate calculations of free-free absorption, the relatively weak dependence of the equipartition and minimum energy estimates on the flux, the uncertainty in other factors like emitting volume (both because of projection

Table 3. Updated equipartition estimates for starbursts

Starburst	D	R	h	ν	S_ν	Bremsstrahlung-scaled ^a			Classical ^b	Revised ^c		References ^d
	(Mpc)	(pc)	(pc)	(GHz)	(Jy)	U_{CR} (eV cm ⁻³)	B_{eq} (μG)	B_{min} (μG)	B_{min} (μG)	B_{eq} (μG)	B_{min} (μG)	
M 82	3.6	250	50	1.0	8.94	940	190	180	160	240	220	(1)
NGC 253 Core	3.5	150	50	1.0	3.0	880	190	180	160	230	220	(1)
NGC 4945	3.7	540	50	1.4	4.2	300	110	100	89	130	130	(2)
NGC 1068 Starburst	13.7	3000	50	1.4	1	86	59	55	47	72	68	(3)
IC 342	4.4	710	50	1.4	2.25	190	87	82	71	110	100	(4)
NGC 2146	12.6	520	50	1.4	1.09	570	150	140	130	190	180	(4)
NGC 3690	42.2	2460	50	1.4	0.66	300	110	100	89	130	130	(4)
NGC 1808	14.2	1030	50	1.4	0.52	200	91	85	73	110	100	(4)
NGC 3079	15.9	190	50	1.4	0.85	2000	280	270	240	350	330	(4)
Arp 220 West	75	100	50	8.4	0.072	9300	610	580	500	750	710	(5)
Arp 220 East	75	100	50	8.4	0.061	8500	590	550	480	720	670	(5)
Arp 193	99	160	50	8.4	0.035	5000	450	420	360	550	520	(4)
Mrk 273	160	160	50	8.4	0.044	9800	630	590	510	770	720	(4)

Columns: D is the distance to the starburst; R is the assumed radius of the starburst disc and h is the assumed midplane-to-edge scale height of the starburst disc, for a total volume of $V = 2\pi R^2 h$; S_ν is the total observed flux density at frequency ν . We take the thermal fraction of the radio emission to be $f_{\text{therm}} = [9(\nu/\text{GHz})^{-0.6} + 1]^{-1}$.

^a: Equipartition energy densities, equipartition magnetic field strengths, and minimum energy magnetic field strengths calculated assuming radio emission is hadronic and with e^\pm lifetimes scaled to the bremsstrahlung loss time, from equations 32, 35, and 36. We assume $X = 3$, $f_{\text{sec}} = 1$, $f_{\text{fill}}^{\text{CR}} = 1$, $\zeta = 1$, and $p = 2.2$.

^b: Classical minimum energy estimate of magnetic field strength from Longair (2010), which assumes $K_0 = 100$, $\nu_{\text{min}} = \nu$, and $\alpha = 0.75$.

^c: Revised equipartition and minimum energy magnetic field strengths from Beck & Krause (2005), which assumes $p = 2.2$ (synchrotron spectral index $\alpha = 0.6$) and $K_0 = 100$.

^d: References – (1): M 82 and NGC 253 radio flux densities from Williams & Bower (2010); (2): NGC 4945 radio flux density and size from Strickland et al. (2004); (3): Radio flux density and approximate radius of NGC 1068 starburst from Wynn-Williams, Becklin, & Scoville (1985); (4): Radio flux densities and radii compiled from Thompson et al. (2006); (5): Distance and sizes of Arp 220’s radio nuclei from Sakamoto et al. (2008); radio flux density from Thompson et al. (2006).

effects and because of the unknown value of the CR filling factor), and the fact that free-free absorption optical depth is expected to be significantly less than 1 at the frequencies we consider (1 - 1.4 GHz for most starbursts, 8.4 GHz for ULIRGs), we ignore it when estimating the magnetic fields and CR energy densities below. However, its effects should be considered carefully when working at low frequencies with the ionization-dominant losses formulae.

Finally, not all of the synchrotron flux comes from the CRs in the starburst interstellar medium. First, some of the radio emission actually comes from individual radio sources like supernovae remnants. The fraction of the flux from these individual sources is only a few percent of the total starburst flux, though, so we ignore it (Lisenfeld & Völk 2000; Lonsdale et al. 2006). Second, not all of the flux comes from the starbursting region itself, where the CRs are accelerated or created through pion productions, and where the magnetic and CR energy densities are thought to be highest. Some comes from the surrounding host galaxy, and additional emission can arise from a “halo” region thought to arise when CR e^\pm are advected away from the galaxy. In the case of NGC 253, about half of the radio flux in fact comes from these regions rather than the starburst core, so we use the core flux only (Heesen et al. 2009; Williams & Bower 2010). Likewise, in NGC 1068, most of the radio flux comes from an active nucleus and its jets, so we use the estimated flux of the starburst alone (Wynn-Williams et al. 1985). We assume in the other cases that all of the synchrotron flux comes from the starburst core, but this can be tested with resolved observations. This is known to be a reasonable approximation for M 82, where the starburst core does actually dominate the total flux at frequencies above 1 GHz (Adebahr et al. 2012).

3.2 New equipartition estimates of B in starbursts

We present new equipartition estimates of the magnetic field strength in selected starburst galaxies in Table 3. These estimates use the bremsstrahlung-dominated formula with $X = 3$, and also assume that $p = 2.2$ and $f_{\text{fill}}^{\text{CR}} = f_{\text{sec}} = \zeta = 1$. For comparison, we also give the results for the classical and revised equipartition formula. Note that we use 8.4 GHz radio data for the ULIRGs Arp 220, Arp 193, and Mrk 273; at these higher frequencies, bremsstrahlung still should be the most important loss (Figure 1, right panel).

The equipartition magnetic field strengths range from 60–600 μG . For M 82 and NGC 253, the estimated magnetic fields are 200 μG , which is entirely in line with broadband modelling of their nonthermal spectra (e.g., de Cea del Pozo et al. 2009a). Klein et al. (1988) and Adebahr et al. (2012) found smaller magnetic field strengths of 50–100 μG , but they assumed

the emission was spread over a larger volume so that the energy densities were smaller. The magnetic field strength estimates for the distant ULIRGs such as Arp 220 are of order half a milliGauss, which is several times smaller than the few milliGauss expected from the linearity of the FIR-radio correlation (Condon et al. 1991; Lacki et al. 2010) and detailed modeling of Arp 220's CR population (Torres 2004). The derived magnetic field strengths depend weakly on $\zeta = U_B/U_{\text{CR}}$, though, with $B \propto \zeta^{2/(5+p)}$ and $U_{\text{CR}} \propto \zeta^{-(1+p)/(5+p)}$ for the bremsstrahlung-loss formulae. At a given radio flux, and if no other assumptions are changed, magnetic field strengths that are 3 times stronger than equipartition require $\zeta \approx 50$ and cosmic ray energy densities 6 times smaller than the values derived for $\zeta = 1$.

However, the most interesting feature of the new equipartition estimates evident in Table 3 is that they are still of the same order as the classical and revised equipartition and minimum-energy estimates.

3.3 Why do previous equipartition formulae seem to work?

If previous equipartition formulae do not apply to starbursts, because of starbursts' strong electron cooling and the presence of secondary e^\pm , why do these formulae give similar results to the corrected formula?

The answer is that the steady-state proton/electron ratio at $\sim \text{GeV}$ energies is not far from the canonical ~ 100 in starbursts, although the reasons for this factor are more subtle than usually realized. The steady-state proton/electron number ratio at any given energy is

$$K = \frac{N_p(E)}{N_e(E)} = \frac{6t_\pi X(p-1)f_{\text{sec}}}{F_{\text{cal}}t_{\text{brems}}} \left(\frac{E_p}{E_{\text{sec}}} \right)^{p-2} \quad (64)$$

In a starburst like M 82 with $F_{\text{cal}} = 0.4$, $p = 2.2$, and $X = 3$ (and therefore $f_{\text{sec}} = 0.73$), we find $K = 120$. Even for $F_{\text{cal}} = 1$, $K = 57$. Therefore, an estimate of the proton energy density that assumes a constant proton/electron ratio of $K = 100$ is actually correct to order of magnitude for 1 GHz observations.

The reasons the assumption works are that:

- While synchrotron and IC strongly suppress the proton/electron ratio at high energy, and while ionization losses strongly suppress the proton/electron ratio at low energy, bremsstrahlung-dominated losses roughly preserve the proton/electron ratio, since $t_\pi/t_{\text{brems}} \approx 1.6$. Although bremsstrahlung is more important at high density, reducing the steady-state electron density (Beck & Krause 2005), the pionic losses also scale with density, reducing the steady-state proton density.
- At 1 GHz, we are observing at intermediate energies in starbursts, a sweet spot where none of the other losses greatly overpowers bremsstrahlung. But while bremsstrahlung may be the strongest individual loss, the other losses combined are more important than bremsstrahlung ($X \approx 3$). As seen in Figure 1, the e^\pm radiative lifetime reaches a maximum for energies observed at GHz frequencies. So the combined effect of the losses is to suppress the proton/electron ratio, but only by a factor of ~ 5 .
- On the other hand, the proton/electron injection ratio is raised by a factor ~ 5 due to the presence of pionic secondaries. The 'dilution' effect arising because secondary e^\pm are injected at much lower energy than the primary protons helps ensure that the secondaries do not swamp the electron population.

This is related to the 'high- Σ_g conspiracy' posited by Lacki et al. 2010 to explain the FIR-radio correlation from normal galaxies to starbursts: radio emission is enhanced by secondaries, but is suppressed by non-synchrotron losses including bremsstrahlung.

A consequence is that if we move to higher or lower electron energy (or observing frequency), X will increase beyond ~ 3 . The coincidence no longer works, and the proton/electron ratio is increased by either ionization losses at low frequency or synchrotron and IC losses at high frequency. At sufficiently low energies ($\lesssim 100 \text{ MeV}$; observed $\lesssim 40 \text{ MHz}$ in the M 82 and NGC 253 starbursts), the pionic secondary spectrum will also subside due to the kinetics of pion production. At low frequencies, such as those observed by LOFAR, both of these effects can increase the proton/electron ratio above that of the hadronic bremsstrahlung-losses case. At a few hundred MHz, where the secondaries are present, the ionization-loss formulae (section 2.5.2) should work since it already takes into account the changing proton/electron ratio. The assumption of a CR proton injection spectrum of the form of a E^{-p} power law also probably starts to break down at proton energies of a GeV as well, with a momentum power law or some non-linear acceleration spectrum likely more appropriate.

However, while these effects formally invalidate the bremsstrahlung-loss dominated formula outside of the frequency range where bremsstrahlung losses dominate (300 MHz to 6 GHz for M 82; 3 GHz to 15 GHz in Arp 220's nuclei), in practice, their effects are relatively benign. From the classical and revised formulae, the proton/electron ratio affects the magnetic field strength estimates only approximately as $K_0^{2/7.2}$: an order of magnitude change in K_0 only results in a factor ~ 2 change in B_{min} or B_{eq} . At low frequencies, the ratio of ionization to bremsstrahlung loss times only goes as $t_{\text{ion}}/t_{\text{brems}} \propto \nu^{1/2}$, and the proton/electron ratio varies similarly. To take an extreme example, M 82's radio flux at 22.5 MHz is 39 Jy (Roger, Costain, & Stewart 1986). Naively putting these values into the bremsstrahlung loss formula (and assuming all of this flux comes from the starburst region) gives $B_{\text{eq}} = 170 \mu\text{G}$ and $B_{\text{min}} = 160 \mu\text{G}$ – values that are within ~ 25 per cent

of those obtained using the formula at 1 GHz. The more formally correct ionization-loss formulae instead give $B_{\text{eq}} = 180 \mu\text{G}$ and $B_{\text{min}} = 140 \mu\text{G}$.

4 CONCLUSION

We rederive minimum energy and equipartition estimates for the magnetic fields for the conditions that prevail in starburst galaxies. In these galaxies, the radio-emitting CR e^\pm population may consist largely of hadronic pp secondaries rather than primaries. Furthermore, strong radiative losses, particularly bremsstrahlung at GeV energies, set the lifetime of the CR e^\pm and affect the proton/electron ratio. Despite these effects, we found that the steady-state proton/electron ratio is probably close to ~ 100 for GHz-emitting e^\pm , assuming that $t_{\text{brems}} \approx 3t_e$. As a result, the classical and revised equipartition formulae give results that are quite similar to ours, despite their different assumptions.

Although we scale the CR e^\pm lifetime to individual losses (particularly bremsstrahlung) to derive an analytical result, it should also be possible to numerically solve for B_{eq} and B_{min} using the full expression for t_{cool} including all losses, if equation 19 is used. However, since the density would not fully cancel out (due to synchrotron and IC losses in the e^\pm lifetime), and because the IC lifetime depends on the radiation energy density, such an estimate would require both the gas density the CRs experience and the radiation energy density.

While our focus has been on starburst galaxies, the formulae above should apply for any radio source where the CR e^\pm population is mostly pionic pp secondaries and is strongly cooled. Pfrommer & Enßlin (2004) already considered the case of radio emission from galaxy clusters, which may come from secondary e^\pm (Dennison 1980) that cool from synchrotron and IC losses off the CMB over the Gyr lifetimes of clusters. Active Galactic Nuclei may also accelerate CR protons when their jets are stalled by surrounding gas (e.g., Alvarez-Muñiz & Mészáros 2004). These protons may interact with the gas to produce pionic secondaries, which may radiate in the radio; the formulae derived here would also apply in such a case.

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